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SHORTER COMMUNICATION

THE EFFECT OF A STEPWISE DISTRIBUTION OF HEAT TRANSFER ON THE COMPRESSIBLE FLOW OVER A FLAT PLATE

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NOMENCLATURE

h, heat-transfer coefficient for step in heat flux;
h_q, heat-transfer coefficient for uniform heat flux;
k, thermal conductivity;
l, length of the unheated portion;
M_∞, free-stream Mach number;
Re, Reynolds number, $\rho_\infty u_\infty x/\mu_\infty$;
x, longitudinal co-ordinate;
y, normal co-ordinate;
δ, boundary-layer thickness;
δ_T, thermal-layer thickness.

INTRODUCTION

THE GENERAL problem of a boundary-layer flow with discontinuous thermal boundary conditions at some point downstream of the origin of the boundary layer has been studied for a variety of special cases and levels of approximation by several authors.† None of these studies has been exact and all have been restricted to incompressible flow; however, the results are commonly used for superposition-integral solutions to problems involving arbitrary distributions of wall temperature or heat flux. The purpose of this note is to present new experimental data for the problem of a step-function distribution of heat transfer for a laminar, supersonic boundary layer, in comparison with the results which were recently reported by Sparrow and Lin [6].

EXPERIMENT

The present measurements were made in the von Kármán Institute Supersonic Wind Tunnel S-1 on symmetric wedge models, shown in Fig. 1. The models were tested under steady-state conditions at free-stream Reynolds numbers of about $1{\cdot}6 \times 10^6$ and 3×10^6 per meter at $M_\infty = 2{\cdot}2$. Heat was dissipated uniformly by Joule effect in a thin layer of silver which was chemically deposited on the surface of the model.

The models used in the experiment were cast from two different insulating materials,‡ with small thermocouple junctions cast in place flush with the surface. The upper and lower surfaces of the models were silver plated, after these surfaces had been sprayed with a thin layer of epoxy to electrically insulate the thermocouples from the silver film. The technique has been reported in detail by Ginoux [7]. Essentially the plating process is that used in silvering mirrors and can be used with a wide variety of materials. Electrical connections to the silver film heaters were made by means of copper electrodes and silver circuit paint, as shown in Fig. 1. The length of the unheated portion was varied from 0 to 65 mm by scoring the silver layers with a series of spanwise lines and isolating the portion upstream of the lines. The four surfaces of the model were powered independently, and symmetry was maintained on the top and bottom surfaces so as to completely eliminate conduction of heat across the model. Furthermore, the rear surfaces compensated for conduction losses to the sting. Therefore, the only conduction of heat within the model was that due to the longitudinal variation of surface temperature, and this conduction was found to be completely negligible except in the immediate vicinity of the step in heat flux.

As well as providing new experimental data in the compressible flow regime, the present tests offer improvements over previous subsonic experiments due to the symmetry of the configuration, the uniformity of the flow field and pressure distribution, and the elimination of complicated effects of leading edge geometry. In addition a number of

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† Work prior to 1952 has been reviewed by Tribus and Klein [1]; subsequent investigations include Gee and Seban [2], Reynolds *et al.* [3], Smith and Shaw [4], Spalding [5], and Sparrow and Lin [6].

‡ CIBA epoxy resin "Araldite" Type D with thermal conductivity $k \simeq 0{\cdot}17$ kcal/h m degC, and Thiokol foaming plastic resin "Rigithane" with $k \simeq 0{\cdot}06$ kcal/h m degC.

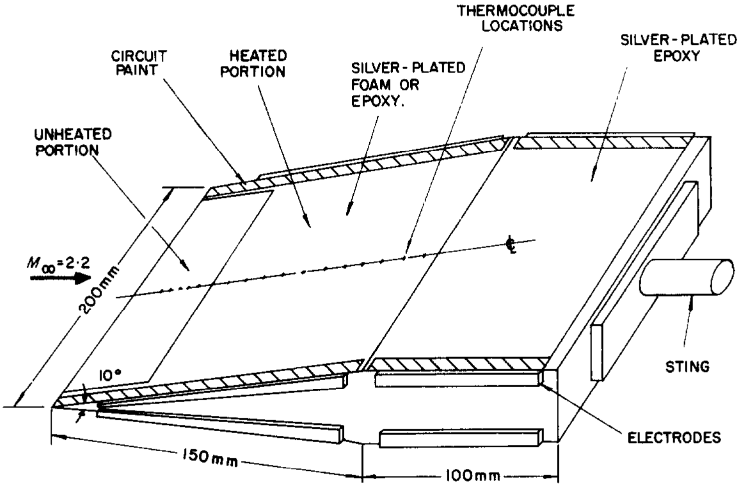

FIG. 1. Sketch of the heat-transfer models.

other uncertainties, such as the absolute values of the properties of air and the absolute readings of the instruments, were cancelled out by measuring both h and h_q and then forming the ratio of the two values. As a further precaution the model temperatures with and without heating were recorded at each set of conditions, and the measured difference was used in computing the heat-transfer coefficients.

DISCUSSION

Smith and Shaw [4], and more recently Sparrow and Lin [6], have analysed the integral form of the incompressible boundary-layer equations for a flat plate with a step in heat flux by assuming cubic velocity and temperature profiles which are similar in the variables (y/δ) and (y/δ_T), respectively. The result is that the effect of the step can be described by a "delay factor", $[1 - l/x]^{-1/3}$, multiplied by the solution for uniform heat flux. Sparrow and Lin imply that this result is also valid for compressible flow. They have also computed the exact solution for the case of uniform heat flux, and the results are the same as the earlier work of Ginoux [7].

Actually the result of Sparrow and Lin regarding the

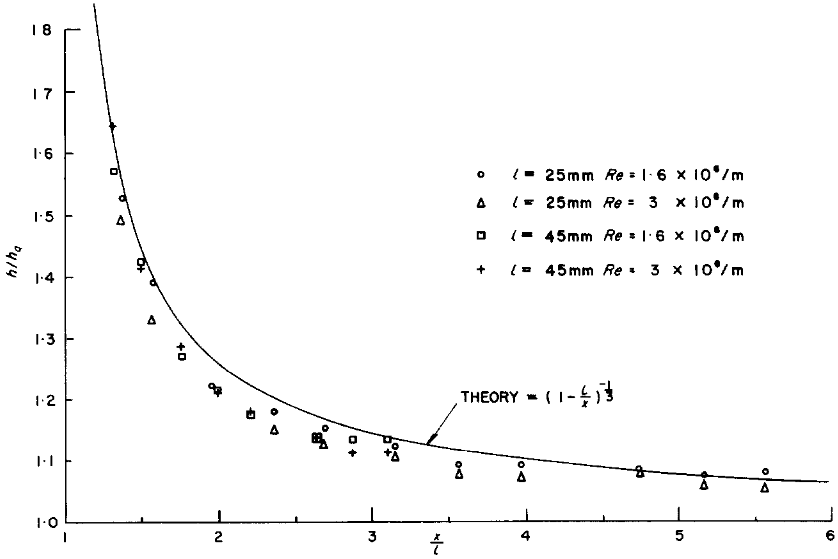

FIG. 2. Non-dimensional heat-transfer results for a step in heat flux.

"delay factor" is more general, with respect to the choice of the velocity and temperature profiles and with respect to the extension to compressible flows. It can be shown [8] that so long as the assumption of similar profiles in Howarth–Dorodnitsyn variables [9] is valid, the profiles can be represented by infinite power series that are completely arbitrary, apart from satisfying the boundary conditions. That is, only the *magnitude* and not the functional dependence of the wall temperature depends upon the shape of the profiles. Any physically acceptable similar profile that can be represented by a power series will give as a first approximation

$$h/h_q = (1 - l/x)^{-1/3},$$

whether the fluid is compressible or not.

Therefore, the task of the experimental investigation is to check the assumption of similar profiles. The non-dimensional test data are shown in Fig. 2, where h/h_q is plotted versus x/l. The test data generally lie on or slightly below the theoretical line $(1 - l/x)^{-1/3}$, with a maximum difference between theory and experiment of about three and a half per cent. There is no discernible difference in the data taken at different free-stream Reynolds numbers and values of l. The data have not been corrected for the effect of conduction within the model. However, this effect is thought to be negligible for $x/l \geqslant 1{\cdot}2$, because these data were unaffected by varying l and unit Reynolds number.

The good agreement between theory and experiment downstream of the step in heat flux indicates that the similar profile assumption is not a crucial one and that the approximate theory is indeed satisfactory. From this agreement the important conclusion can be drawn, that heat transfer on a flat plate with a step in heat flux can be described in terms of well-known, exact solutions and the simple "delay factor" which is a function only of x/l. Finally, it should be pointed out that the delay factor decays rapidly downstream of the discontinuity and amounts to less than 10 per cent for $x/l > 4$.

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